Chapter 1 to 4 End test

/45 Marks

Name:

1. Solve the equation $|5 - 3x| = 10$.

[3]

$5-3x=10$ or $5-3x=-10$

$-3x=5$ $\quad$ $-3x=-15$

$x=-\frac{5}{3}$ $\quad$ $x=5$

$=-1\frac{2}{3}$

2. The polynomial $p(x)$ is $x^4 - 2x^3 - 3x^2 + 8x - 4$.
   a. Show that $p(x)$ can be written as $(x-1)(x^3 - x^2 - 4x + 4)$.

[2]

$x^4 - x^3 - 4x^2 + 4x - x^3 + x^2 + 4x - 4$

$= x^4 - 2x^3 - 3x^2 + 8x - 4$

$= p(x)$ (shown)

   b. Hence write $p(x)$ as a product of its linear factors, showing all your working.

$p(x) = (x-1)(x^3 - x^2 - 4x + 4)$

[4]

let $f(x) = x^3 - x^2 - 4x + 4$

$f(1) = 1 - 1 - 4 + 4$

$= 0$

$(x-1)$ is a factor of $f(x)$.

$$\begin{array}{r|l} & x^2 - 4 \\ \hline x-1 & x^3 - x^2 - 4x + 4 \\ & -(x^3 - x^2) \\ \hline & -4x + 4 \\ & -(-4x + 4) \\ \hline & 0 \end{array}$$

$p(x) = (x-1)(x-1)(x^2-4)$

$= (x-1)(x-1)(x-2)(x+2)$

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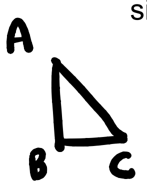
3. **Do not use a calculator in this question.**

In this question, all lengths are in centimetres.

A triangle ABC is such that angle$B = 90°$, $AB = 5\sqrt{3} + 5$ and $BC = 5\sqrt{3} - 5$. Find , in its simplest surd form, the length of AC.



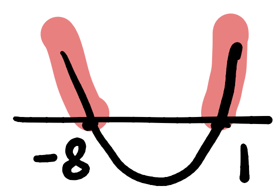
[4]

$$AC^2 = AB^2 + BC^2$$
$$= (5\sqrt{3} + 5)^2 + (5\sqrt{3} - 5)^2$$
$$= 75 + \cancel{50\sqrt{3}} + 25 + 75 - \cancel{50\sqrt{3}} + 25$$
$$= 200$$
$$AC = \sqrt{200}$$
$$= 10\sqrt{2} \text{ cm}$$

4. Solve the inequality $(2 - x)(x + 9) < 10$.

[4]

$$2x + 18 - x^2 - 9x < 10$$
$$-x^2 - 7x + 18 - 10 < 0$$
$$-x^2 - 7x + 8 < 0$$
$$x^2 + 7x - 8 > 0$$
$$(x + 8)(x - 1) > 0$$
$$x < -8 \text{ or } x > 1$$



5. Simplify $\frac{4-3\sqrt{6}}{\sqrt{3}+\sqrt{2}}$ giving your answer in the form $p\sqrt{3} + q\sqrt{2}$, where *p* and *q* are integers.

[4]

$$\frac{4 - 3\sqrt{6}}{\sqrt{3} + \sqrt{2}} \times \frac{(\sqrt{3} - \sqrt{2})}{(\sqrt{3} - \sqrt{2})}$$
$$= \frac{4\sqrt{3} - 4\sqrt{2} - 3\sqrt{18} + 3\sqrt{12}}{3 - 2}$$
$$= 4\sqrt{3} - 4\sqrt{2} - 9\sqrt{2} + 6\sqrt{3}$$

$$10\sqrt{3} - 13\sqrt{2} \;\#$$

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6. Given that $\frac{p^{\frac{1}{3}}q^{\frac{-1}{2}}r^{\frac{3}{2}}}{p^{\frac{-2}{3}}\sqrt{(qr)^5}} = p^a q^b r^c$, find the value of each of the integers a, b and c.

[3]

$$\frac{p^{1/3}q^{-1/2}r^{3/2}}{p^{-2/3}q^{5/2}r^{5/2}} = p^{1/3+2/3}q^{-1/2-5/2}r^{3/2-5/2}$$

$$= p^{3/3}q^{-6/2}r^{-2/2}$$

$$= p^{1}q^{-3}r^{-1}$$

$a = 1$, $b = -3$, $c = -1$

7. The function f is defined by $f(x) = 2 - \sqrt{x+5}$ for $-5 \le x < 0$.

(i) Write down the range of f.

[2]

$f(-5) = 2 - \sqrt{0} = 2$

$f(0) = 2 - \sqrt{5}$

$2 - \sqrt{5} < y \leq 2$

(ii) Find $f^{-1}(x)$ and state its domain and range.

[4]

$y = 2 - \sqrt{x+5}$

$x = 2 - \sqrt{y+5}$

$\sqrt{y+5} = 2 - x$

$y + 5 = (2-x)^2$

$y = (2-x)^2 - 5$

$f^{-1}(x) = (2-x)^2 - 5$

$2 - \sqrt{5} < x \leq 2$

$-5 \leq y < 0$

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The function g is defined by $g(x) = \frac{4}{x}$ for $-5 \leq x < -1$.

(iii) Solve $fg(x) = 0$. [3]

$g(x) = f^{-1}(0)$

$\frac{4}{x} = (2-0)^2 - 5$

$= 4-5$

$= -1$

$\frac{4}{x} = -1$

$x = -4$

8. (i) Express $4x^2 + 8x - 5$ in the form $p(x+q)^2 + r$, where *p*, *q* and *r* are constants to be found. [3]

$4x^2+8x-5 = p(x^2+2qx+q^2)+r$

$= px^2 + 2pqx + pq^2 + r$

$p=4$, $2pq=8$, $pq^2+r=-5$

$8q = 8$ $4+r=-5$

$q=1$ $r=-9$

$4x^2+8x-5 = 4(x+1)^2-9$

(ii) State the coordinates of the vertex of $y = \left|4x^2 + 8x - 5\right|$. [2]

$(-1,-9)$

vertex $= (-1, 9)$

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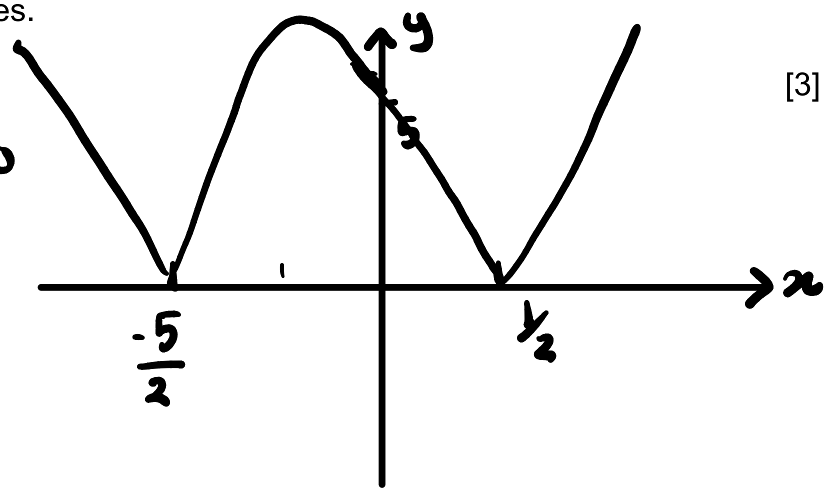
(iii) Sketch the graph of $y = |4x^2 + 8x - 5|$, showing the coordinates of the points where the curve meets the axes. [3]

$x = 0,\ y = -5$

$y = 0,\ 4x^2 + 8x - 5 = 0$

$x = \frac{1}{2} \quad x = -\frac{5}{2}$



9. Find the values of *a* for which the line $y = ax + 9$ intersects the curve $y = -2x^2 + 3x + 1$ at 2 distinct points. [4]

$b^2 - 4ac > 0$

$ax + 9 = -2x^2 + 3x + 1$

$ax + 9 + 2x^2 - 3x - 1 = 0$

$2x^2 + ax - 3x + 8 = 0$

$a = 2,\ b = a - 3,\ c = 8$

$b^2 - 4ac > 0$

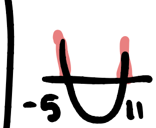
$(a-3)^2 - 4(2)(8) > 0$

$a^2 - 6a + 9 - 64 > 0$

$a^2 - 6a - 55 > 0$

$+5$, -11

$(a+5)(a-11) > 0$



$a > 11$ or $a < -5$